Government of Western Australia
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Mathematics Methods

ATAR course

Year 12 syllabus

IMPORTANT INFORMATION

This syllabus is effective from 1 January 2016.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the School Curriculum and Standards Authority on a cyclical basis, typically every five years.

Content

Overview of mathematics courses

There are six mathematics courses, three General and three ATAR. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The Western Australian Certificate of Education (WACE) examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a General course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a General course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the WACE. It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of the Mathematics Methods ATAR course are calculus and statistics. They include, as necessary prerequisites, studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons, this course provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, this course is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of the Mathematics Methods course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students' learning of this course. These strands are Understanding, Fluency, Problem-solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of this course. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem-solving throughout this course.

The Mathematics Methods ATAR course is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the Year 7–10 Australian curriculum from the strands Number and Algebra, Measurement and Geometry, and Statistics and Probability. In this course, there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3, where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Aims

The Mathematics Methods ATAR course aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 12 syllabus is divided into two units which are delivered as a pair. The notional time for the pair of units is 110 class contact hours.

Organisation of content

Unit 3

Contains the three topics:

- Further differentiation and applications
- Integrals
- Discrete random variables.

The study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

Unit 4

Contains the three topics:

- The logarithmic function
- Continuous random variables and the normal distribution
- Interval estimates for proportions.

The logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit, students are introduced to one of the most important parts of statistics, namely, statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students will be taught this course with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of the course. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Representation of the general capabilities

The general capabilities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities into the teaching and learning program for the Mathematics Methods ATAR course. The general capabilities are not assessed unless they are identified within the specified unit content.

Literacy

Literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy

Students who undertake this course will continue to develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the ever-increasing demands of the information age, developing the skills of critical evaluation of numerical information. Students will enhance their numerical operation skills by solving of practical problems using the calculus of trigonometric, exponential and logarithmic functions, and by working with discrete and continuous random variables and associated data distributions.

Information and communication technology capability

Students use information and communication technology (ICT) both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved, such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability

Students develop personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making. The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for their decision making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding

Students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results, and the social responsibility associated with teamwork and attribution of input. The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols, but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Representation of the cross-curriculum priorities

The cross-curriculum priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate the priorities into the teaching and learning program for the Mathematics Methods ATAR course. The cross-curriculum priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 3

Unit description

The study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 3.1: Further differentiation and applications (20 hours)

Exponential functions

3.1.1 estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$, using technology, for various values of $a > 0$

3.1.2 identify that e is the unique number a for which the above limit is 1

3.1.3 establish and use the formula $\frac{d}{dx}(e^x) = e^x$

3.1.4 use exponential functions of the form Ae^{kx} and their derivatives to solve practical problems

Trigonometric functions

3.1.5 establish the formulas $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions

3.1.6 use trigonometric functions and their derivatives to solve practical problems

Differentiation rules

3.1.7 examine and use the product and quotient rules

3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions

3.1.9 apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x\sin x$, $e^{-x}\sin x$ and $f(ax-b)$

The second derivative and applications of differentiation

3.1.10 use the increments formula: $\delta y \approx \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x

3.1.11 apply the concept of the second derivative as the rate of change of the first derivative function

3.1.12 identify acceleration as the second derivative of position with respect to time

3.1.13 examine the concepts of concavity and points of inflection and their relationship with the second derivative

3.1.14 apply the second derivative test for determining local maxima and minima

3.1.15 sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection

3.1.16 solve optimisation problems from a wide variety of fields using first and second derivatives

Topic 3.2: Integrals (20 hours)

Anti-differentiation

3.2.1 identify anti-differentiation as the reverse of differentiation

3.2.2 use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals

3.2.3 establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$

3.2.4 establish and use the formula $\int e^x dx = e^x + c$

3.2.5 establish and use the formulas $\int \sin x\, dx = -\cos x + c$ and $\int \cos x\, dx = \sin x + c$

3.2.6 identify and use linearity of anti-differentiation

3.2.7 determine indefinite integrals of the form $\int f(ax-b)dx$

3.2.8 identify families of curves with the same derivative function

3.2.9 determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$

Definite integrals

3.2.10 examine the area problem and use sums of the form $\sum_i f(x_i)\, \delta x_i$ to estimate the area under the curve $y = f(x)$

3.2.11 identify the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\, \delta x_i$

3.2.12 interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$

3.2.13 interpret $\int_a^b f(x)dx$ as a sum of signed areas

3.2.14 apply the additivity and linearity of definite integrals

Fundamental theorem

3.2.15 examine the concept of the signed area function $F(x) = \int_a^x f(t)dt$

3.2.16 apply the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically

3.2.17 develop the formula $\int_a^b f'(x)dx = f(b) - f(a)$ and use it to calculate definite integrals

Applications of integration

3.2.18 calculate total change by integrating instantaneous or marginal rate of change

3.2.19 calculate the area under a curve

3.2.20 calculate the area between curves

3.2.21 determine displacement given velocity in linear motion problems

3.2.22 determine positions given linear acceleration and initial values of position and velocity.

Topic 3.3: Discrete random variables (15 hours)

General discrete random variables

3.3.1 develop the concepts of a discrete random variable and its associated probability function, and their use in modelling data

3.3.2 use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable

3.3.3 identify uniform discrete random variables and use them to model random phenomena with equally likely outcomes

3.3.4 examine simple examples of non-uniform discrete random variables

3.3.5 identify the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases

3.3.6 identify the variance and standard deviation of a discrete random variable as measures of spread, and evaluate them using technology

3.3.7 examine the effects of linear changes of scale and origin on the mean and the standard deviation

3.3.8 use discrete random variables and associated probabilities to solve practical problems

Bernoulli distributions

3.3.9 use a Bernoulli random variable as a model for two-outcome situations

3.3.10 identify contexts suitable for modelling by Bernoulli random variables

3.3.11 determine the mean p and variance $p(1-p)$of the Bernoulli distribution with parameter p

3.3.12 use Bernoulli random variables and associated probabilities to model data and solve practical problems

Binomial distributions

3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial

3.3.14 identify contexts suitable for modelling by binomial random variables

3.3.15 determine and use the probabilities $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ associated with the binomial distribution with parameters n and p; note the mean np and variance $np(1-p)$ of a binomial distribution

3.3.16 use binomial distributions and associated probabilities to solve practical problems

Unit 4

Unit description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In the Mathematics Methods ATAR course, statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit content

This unit builds on the content covered in Unit 3.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 4.1: The logarithmic function (18 hours)

Logarithmic functions

4.1.1 define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$

4.1.2 establish and use the algebraic properties of logarithms

4.1.3 examine the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$

4.1.4 interpret and use logarithmic scales

4.1.5 solve equations involving indices using logarithms

4.1.6 identify the qualitative features of the graph of $y = \log_a x$ $(a > 1)$, including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x - c)$

4.1.7 solve simple equations involving logarithmic functions algebraically and graphically

4.1.8 identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems

Calculus of the natural logarithmic function

4.1.9 define the natural logarithm $\ln x = \log_e x$

4.1.10 examine and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$

4.1.11 establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$

4.1.12 establish and use the formula $\int \frac{1}{x} dx = \ln\ x\ + c$, for $x > 0$

4.1.13 determine derivatives of the form $\frac{d}{dx}(\ln f(x))$ and integrals of the form $\int \frac{f'(x)}{f(x)} dx$, for $f\ (x) > 0$

4.1.14 use logarithmic functions and their derivatives to solve practical problems

Topic 4.2: Continuous random variables and the normal distribution (15 hours)

General continuous random variables

4.2.1 use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable

4.2.2 examine the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts

4.2.3 identify the expected value, variance and standard deviation of a continuous random variable and evaluate them using technology

4.2.4 examine the effects of linear changes of scale and origin on the mean and the standard deviation

Normal distributions

4.2.5 identify contexts, such as naturally occurring variation, that are suitable for modelling by normal random variables

4.2.6 identify features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution

4.2.7 calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems

Topic 4.3: Interval estimates for proportions (22 hours)

Random sampling

4.3.1 examine the concept of a random sample

4.3.2 discuss sources of bias in samples, and procedures to ensure randomness

4.3.3 use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli

Sample proportions

4.3.4 examine the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ of the sample proportion $\hat{p}$

4.3.5 examine the approximate normality of the distribution of $\hat{p}$ for large samples

4.3.6 simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$ where the closeness of the approximation depends on both n and p

Confidence intervals for proportions

4.3.7 examine the concept of an interval estimate for a parameter associated with a random variable

4.3.8 use the approximate confidence interval $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution

4.3.9 define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence

4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p

School-based assessment

The Western Australian Certificate of Education (WACE) Manual contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Methods ATAR Year 12 syllabus and the weighting for each assessment type.

Assessment table – Year 12

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem-solving skills and algorithms. Response tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to open-ended questions that require responses at the highest level of conceptual thinking. Students can be asked questions of an investigative nature for which they may need to communicate findings, generalise, or make and test conjectures. Typically conducted at the end of each semester and/or unit and reflecting the examination design brief for this syllabus.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units.

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units

- each assessment type must be included at least twice
- the response type must include a minimum of two tests.

The set of assessment tasks must provide a representative sampling of the content for Unit 3 and Unit 4.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units. The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Methods ATAR Year 12 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the WACE Manual for further information about the use of a ranked list in the process of assigning grades.

WACE examination

All students enrolled in the Mathematics Methods ATAR Year 12 course are required to sit the WACE examination. The examination is based on a representative sampling of the content for Unit 3 and Unit 4. Details of the WACE examination are prescribed in the examination design brief on the following page.

Refer to the WACE Manual for further information.

Examination design brief – Year 12

This examination consists of two sections.

Section One: calculator-free

Time allowed

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Permissible items

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Additional information

Changeover period during which the candidate is not permitted to work: up to 15 minutes

Section Two: Calculator-assumed

Time allowed

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Permissible items

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Provided by the supervisor

A formula sheet

Additional information

It is assumed that candidates sitting this examination have a calculator with CAS capabilities for Section Two. The examination assesses the syllabus content areas using the following percentage ranges. These apply to the whole examination rather than individual sections.

Content area	Percentage of exam
Calculus and logarithmic functions	50–60%
Random variables (DRV's and CRV's)	25–30%
Interval estimates	15–20%

The candidate is required to demonstrate knowledge of mathematical facts, conceptual understandings, use of algorithms, use and knowledge of notation and terminology, and problem-solving skills.

Questions can require the candidate to investigate mathematical patterns, make and test conjectures, generalise and prove mathematical relationships. Questions can require the candidate to apply concepts and relationships to unfamiliar problem-solving situations, choose and use mathematical models with adaptations, compare solutions and present conclusions. A variety of question types that require both open and closed responses can be included.

Instructions to candidates indicate that, for any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Section	Supporting information
Section One: calculator-free 35% of the total examination 5–10 questions Working time: 50 minutes	Questions examine content and procedures that can reasonably be expected to be completed without the use of a calculator, i.e. without undue emphasis on algebraic manipulations or time-consuming calculations. The candidate is required to provide answers that include calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media.
Section Two: calculator-assumed 65% of the total examination 8–13 questions Working time: 100 minutes	Questions examine content and procedures for which the use of a calculator is assumed. The candidate can be required to provide answers that include calculations, tables, graphs, interpretation of data, descriptions and conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media. The candidate can be required to investigate theoretical situations involving mathematical concepts and relationships, for which they need to generalise, construct proofs and make conjectures. The candidate can be required to solve problems from unfamiliar situations, choosing and using mathematical models with adaptations where necessary, comparing their solutions with the situations concerned, and then presenting their findings in context.

Appendix 1 – Grade descriptions Year 12

A	**Identifies and organises relevant information** Identifies and organises relevant information that is dense and scattered for complex problems involving a series of steps or processes, for example, correctly identifies a problem as a specific case; identifies a binomial model and defines the relevant expression or recognises a representation of Bernoulli trials. Incorporates information that is needed to define equations from text and diagrams, for example, in problems involving determination of the maximum of a quantity; or a descriptive passage containing information about random variables and associated probabilities to solve practical problems.
	Chooses effective models and methods and carries the methods through correctly Chooses an effective method and correctly carries the method through in an extended response, for example, uses an appropriate probability distribution to calculate a required probability value. Generalises and extends models from previous part(s) of the question, for example, chooses the correct equations and correctly applies the derivative to find the optimum result.
	Obeys mathematical conventions and attends to accuracy Obeys conventions and attends to accuracy when calculating compound probability, for example, using a tree diagram or the multiplication principle to calculate a probability. Obeys mathematical conventions when using piece-wise functions to define a probability density function or to approximate the interval estimate of a sample proportion.
	Links mathematical results to data and contexts to reach reasonable conclusions Uses the second derivative of a function to determine the nature of the concavity of the graph and hence to locate points of inflection. Makes links between displacement, velocity and acceleration of a particle to determine the distance travelled in a given time period.
	Communicates mathematical reasoning, results and conclusions Sets out mathematical reasoning, results and conclusions when solving practical problems involving probability distributions. Sets out mathematical reasoning, results and conclusions when using functions and their derivatives to solve practical problems.

B	**Identifies and organises relevant information** Identifies and organises relevant information that is dense and scattered for less complex problems, such as those involving only a few steps or processes, for example, identifies variables in a given diagram or for a related rates problem, draws a diagram with appropriate variables; identifies probability values from a descriptive passage or the definite integral required to determine the area between two curves.
	Chooses effective models and methods and carries the methods through correctly Chooses an effective method or variables then correctly carries the method through for problems that contains only a few steps, for example, uses the incremental formula to obtain an expression for percentage change; and defines the parameters of the Normal or Binomial distributions and carries the correct calculations through.
	Obeys mathematical conventions and attends to accuracy Obeys conventions and attends to accuracy, for example, when using the increments formula; drawing the graphs of probability density functions accurately; using the second derivative to check for maximum or minimum; with units involving related rates, and with respect to units when accurately applying the chain rule.
	Links mathematical results to data and contexts to reach reasonable conclusions Uses the graph of the derivative function to determine the nature of the turning points of the original function. Links a calculated result to draw appropriate conclusions, for example, applies the incremental formula, interprets the result and draws the correct conclusion about the validity of the percentage increase or decrease.
	Communicates mathematical reasoning, results and conclusions Sets out mathematical reasoning, results and conclusions when working with extended optimisation problems, for example, proves differential equations with multiple terms or provides logical working steps for calculating related rates.

C	**Identifies and organises relevant information** Identifies and organises relevant information that is grouped together and is narrow in scope, for example, the fundamental theorem as the derivative of an integral; the relevant definite integral to determine the area between two curves; information relating to a probability density function.
	Chooses effective models and methods and carries the methods through correctly Chooses an effective integral to define the area between two graphs, and correctly carries through, finding the area. Chooses an effective method and correctly evaluates the parameter k defined in the differential equation $\frac{dP}{dt} = kP$. Chooses and uses the correct parameters when calculating a normal probability value $P(X \leq x)$ given μ and σ
	Obeys mathematical conventions and attends to accuracy Applies basic conventions for diagrams and graphs. Applies rules and checks for accuracy when evaluating integrals; and applies the chain rule correctly. Obeys conventions and uses correct notation, for example, for differentiation and with probability distributions. Checks for accuracy of calculations, including those where technology is used, for example, to evaluate an integral.
	Links mathematical results to data and contexts to reach reasonable conclusions Uses the graph of the derivative function to locate the turning points of the original function. Links the first derivative of a displacement function to velocity, including the appropriate units, and relates acceleration using the second derivative. Recognises specified conditions in short responses and rejects solutions to optimisation problems because they are outside the domain, for example, $t < 0$.
	Communicates mathematical reasoning, results and conclusions Communicates mathematically when naming probability distributions, for example, defines the binomial distribution using p. Uses the second derivative of a function to determine the nature of the turning points of a function.

D	**Identifies and organises relevant information** Identifies the range of possible scores in a probability experiment. Identifies and organises the relevant definite integral, from a simple diagram, to use for the calculation of area under a curve. Finds the intersection of two graphs. Calculates a simple probability value using a probability density function, and recognises that sum of probabilities equals one.
	Chooses effective models and methods and carries the methods through correctly Applies mathematical methods, for example, differentiation and integration, in practised ways. Interprets selections in practised ways and calculates each selection $\binom{n}{r}$ accurately. Calculates the probability of a discrete random variable using a table of values.
	Obeys mathematical conventions and attends to accuracy Uses technology to evaluate an integral but gives only the answer. Enters data correctly into a calculator but tends to give numerical answers without working.
	Links mathematical results to data and contexts to reach reasonable conclusions Attends to units in short responses and rounds to suit the context when required, for example, number of batteries expressed as a whole number.
	Communicates mathematical reasoning, results and conclusions Uses basic symbols and notation, for example, ≤, $\bar{x}$, $

E	Does not meet the requirements of a D grade.

Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

Unit 3

Further differentiation and applications	
Composition of functions	If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$.
Chain rule	The chain rule relates the derivative of the composite of two functions to the functions and their derivatives. If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$, and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$
Euler's number	Euler's number e is an irrational number whose decimal expansion begins $e = 2.7182818284590452353602874713527 \cdots$ It is the base of the natural logarithms, and can be defined in various ways, including: $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$
Point of inflection	A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P.
Product rule	The product rule relates the derivative of the product of two functions to the functions and their derivatives. If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$, and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$
Quotient rule	The quotient rule relates the derivative of the quotient of two functions to the functions and their derivatives If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Second derivative test	According to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$.
Integrals	
Additivity property of definite integrals	The additivity property of definite integrals refers to 'addition of intervals of integration': $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

Anti-differentiation	An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$. The process of solving for anti-derivatives is called anti-differentiation. Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$.
The fundamental theorem of calculus	The fundamental theorem of calculus relates differentiation and definite integrals. It has two forms: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$.
The linearity property of anti-differentiation	The linearity property of anti-differentiation is summarised by the equations: $\int kf(x)dx = k\int f(x)dx$ for any constant k and $\int\left(f_1(x) + f_2(x)\right)dx = \int f_1(x)dx + \int f_2(x)\,dx$ for any two functions $f_1(x)$ and $f_2(x)$ Similar equations describe the linearity property of definite integrals: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any constant k and $\int_a^b\left(f_1(x) + f_2(x)\right)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$ for any two functions $f_1(x)$ and $f_2(x)$.
Discrete random variables	
Bernoulli random variable	A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.
Bernoulli trial	A Bernoulli trial is a chance experiment with possible outcomes, typically labelled 'success' and 'failure'.
Effect of linear change	The effects of linear changes of scale and origin on the mean and variance of a random variable are summarised as follows: If X is a random variable and $Y = aX + b$, where a and b are constants, then $E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$.
Expected value	The expected value $E(X)$ of a random variable X is a measure of the central tendency of its distribution. If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and $p_i = P(X = x_i)$. If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X.
Mean of a random variable	The mean of a random variable is another name for its expected value.

Point and interval estimates	In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probablity or population parameter. If the estimate is a single number, this number is called a point estimate. An interval estimate is an interval derived from the sample that, in some sense, is likely to contain the parameter. A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.
Probability distribution	The probability distribution of a discrete random variable is the set of probabilities for each of its possible values.
Random variable	A random variable is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year. A discrete random variable is one whose possible values are the counting numbers $0,1,2,3,\cdots$, or form a finite set, as in the first two examples. A continuous random variable is one whose set of possible values are all of the real numbers in some interval.
Standard deviation of a random variable	The standard deviation of a random variable is the square root of its variance.
Uniform discrete random variable	A uniform discrete random variable is one whose possible values have equal probability of occurrence. If there are n possible values, the probability of occurrence of any one of them is $\frac{1}{n}$.
Variance of a random variable	The variance $Var(X)$ of a random variable X is a measure of the 'spread' of its distribution. If X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value. If X is continuous, $Var(X) = \int_{-\infty}^{\infty}(x-\mu)^2 p(x)dx$.

Unit 4

The logarithmic function

Algebraic properties of logarithms	The algebraic properties of logarithms are the rules: • $\log_a(xy) = \log_a x + \log_a y$ • $\log_a x^n = n \log_a x$ • $\log_a \frac{1}{x} = -\log_a x$ • $\log_a 1 = 0$, for n any integer and for positive real numbers x, y and a.

Continuous random variables and the normal distribution	
Probability density function	The probability density function of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\,dx$.
Quantile	A quantile t_α for a continuous random variable X is defined by $P(X < t_\alpha) = \alpha$, where $0 < \alpha < 1$. The 75$^{\text{th}}$ percentile of X is the quantile corresponding to $\alpha = 0.75$: $P(X < t_\alpha) = 0.75$, Thus the 0·75 quantile (also called the 75th percentile or upper quartile) is the score that 75% of the population lies below.
Triangular continuous random variable	A triangular continuous random variable X is one whose probability density function $p(x)$ has a graph with the shape of a triangle.
Uniform continuous random variable	A uniform continuous random variable X is one whose probability density function $p(x)$ has constant value on the range of possible values of X. If the range of possible values is the interval $[a, b]$, then $p(x) = \frac{1}{b-a}$ if $a \le x \le b$ and $p(x) = 0$ otherwise.

Interval estimates for proportions	
Central limit theorem	There are various forms of the central limit theorem, a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows: "If $\bar{X}$ is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution." In the special case where X is a Bernoulli random variable with parameter p, $\bar{X}$ is the sample proportion $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{\frac{p(1-p)}{n}}$. In this case, the central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.
Margin of error	The margin of error of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.
Level of confidence	The level of confidence associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.